SCUOLA DI INGEGNERIA INDUSTRIALE
E DELL'INFORMAZIONE

Course: Dynamics of electrical machine and drives - 10 CFU

Laurea magistrale in Automation and Control Engineering

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Tramway "Carelli 1928"

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1. Abstract

Tramway vehicles "Carelli 1928" by ATM (Azienda Trasporti Milanesi S.p.A.) are moved by four DC motors in separately excitation. This electromechanical configuration, central to the Series 1500 cars, represents a milestone in 20th-century transit engineering that remains operational within Milan's modern urban fabric.
This document analyses the operational characteristics of these separately excited motors. The study further explores:

- The identification of the parameters of the system

- The design and simulation for a speed control, covering the 10km track

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2. Model identification

2.1 Motor parameters

The problem is to analyse and control a separately excited DC motor used to move an ATM tramway "Carelli 1928".

- Excitation rated voltage: $V_{er} = 60\ V$
- Excitation rated current: $I_{er} = 5\ A$
- Excitation resistance: $R_e = 12\ \Omega$
- Excitation time constant: $\tau_e = 0{,}1\ s$
- Line voltage: $V_{DC} = 600\ V$
- Rated power of each motor: $P_r = 21\ kW$
- Maximum speed of the vehicle: $v_{max} = 42\frac{km}{h}$
- Rated speed of the motor: $\Omega_r = 970\ rpm$
- Overall rated current: $I_{tot} = 156\ A$
- Torque constant: $\mathrm{K_s} = 1{,}06\frac{Nm}{A^2}$
- Armature resistance: $R_a = 0{,}39\ \Omega$
- Armature circuit time constant: $\tau_a = 10\ ms$

2.2 Other parameters

The dynamic model of the tramway is described by these parameters:

- Mass of the vehicle (no load): $m_T = 15.000\ kg$
- Maximum loading capacity: 130 passengers
- Average mass of a passenger: $m_{pass} = 80\ kg$
- Diameter of the wheel: $d = 680\ mm$
- Gearbox ratio (motor-to-wheels): $\rho = \frac{13}{74}$
- Friction coefficient: $\beta = 0{,}81\ Nms$

2.3 Kinematic characteristic

I have considered the longitudinal trajectory of the tramway since the requirement is the traction control.
The tram's speeds are described in the following table:

Distance [km]	Slope %	Speed
0-1	0	$v_r/2$
1-3	0	v_r
3-4	5	v_r
4-6	0	v_{max}
6-8	0	v_r
8-9	-5	v_r
9-10	0	$v_r/2$

Table 1 - Kinematic characteristics

3. Modelling of the physical system

In Figure 1 is described the control scheme for a separately excited DC motor. Each subsystem will be discussed below.
The study is conducted on the electrical model of the motor, which has a faster dynamic, and the mechanical motor.
The change of the mechanical load, due to trajectory, will be considered as a disturbance.

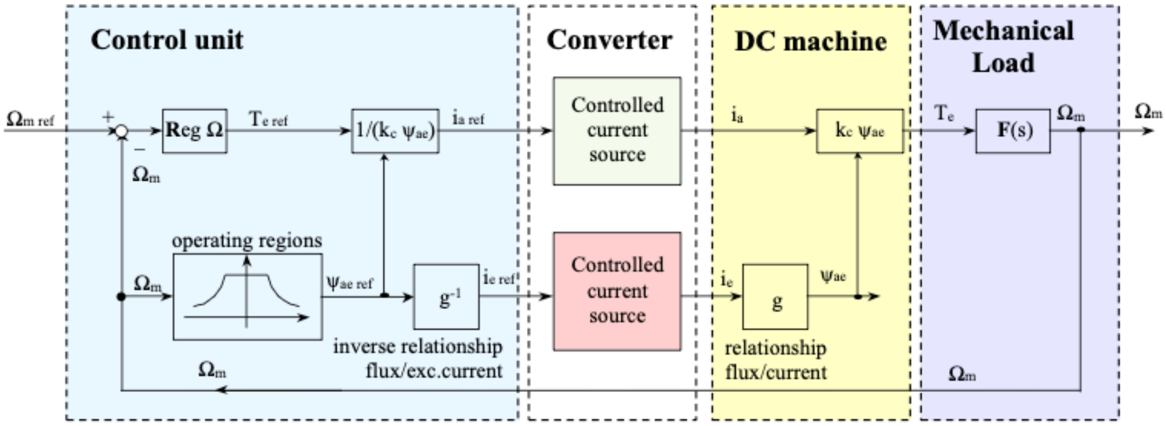


Figure 1 - Speed control scheme of DC machine

A critical step in the model identification concerns the interpretation of the machine constant. Although the technical specifications provide a torque constant $K_t = 1.06\ Nm/A$, a rigorous application of Kirchhoff's Voltage Law (KVL) to the armature circuit ($V = R_a * I_a + e$) reveals that this value must be treated as the total machine constant Ks. Given the nominal operating point ($V = 600\ V$, $I_a = 156\ A$, $R_a = 0.39\ \Omega$), the motor must generate a back-electromotive force (e.m.f.) of approximately 539 V.
By applying the fundamental relation $e = K_s \cdot I_e \cdot \omega$, it is evident that only Ks yields a result consistent with the physical constraints at the rated speed of $101.6\ rad/s$ and $5\ A$ excitation. Any other interpretation would result in a significant voltage mismatch, making the achievement of the rated performance physically impossible within the 600 V line limit.

3.1 Electrical model

This section describes the modelling and control strategy for a separately excited DC motor, focusing on the decoupling of electrical and mechanical dynamics.
The Figure 3 illustrates the standard simulation diagram of a PM DC motor.
The equivalent moment of inertia J_{eq} referred to the motor shaft accounts exclusively for the translational inertia of the vehicle reflected through the kinematic chain.
The following contributions are neglected:

- Rotor inertia of the four DC motors (typical order 0.1–1 kg·m² each, referred to the motor shaft): < 5 % of J_{eq};
- Wheelset rotational inertia: for four axles with estimated $J_{wheel} \approx 25\ kg \cdot m^2$ per axle, reflected by $\rho^2 \approx 4 \cdot 25 \cdot \left(\frac{13}{74}\right)^2 \approx 3.1\ kg \cdot m^2$
- Gearbox and driveshaft inertia: negligible (< 0.5 %).

Total omitted inertia is bounded by ≈ 9 % of J_{eq}, which produces a bandwidth shift of the same order on the speed loop.
This is acceptable given the target $\omega_m = 2\ rad/s$ and the dominant role of disturbance torque on transient response.

The electrical plant is defined by the transfer function

$$G_i(s) = \frac{1}{R + sL}$$

And the mechanical load by:

$$G_\omega(s) = \frac{1}{\beta + sJ}$$

A key feature of the model is the inherent coupling caused by the back-electromotive force (back-EMF) $e = K\omega$.
This signal acts as a feedback loop that subtracts from the applied voltage v, meaning the armature current i is directly influenced by the motor speed ω, complicating the independent design of the controller.

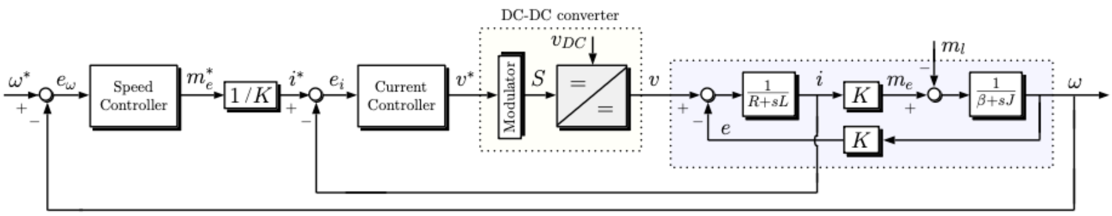


Figure 2 - Simulation diagram of PMDC motor

To simplify the control system, is needed to introduce a decoupling strategy using a feedforward term.
By adding an estimated back-EMF compensation $e\wedge = K\omega\wedge$ to the voltage reference v^* the physical effect of e is cancelled out.
This feedforward action effectively "unties" the electrical dynamics from the mechanical ones, allowing the current and speed loops to be tuned as separate, independent systems.

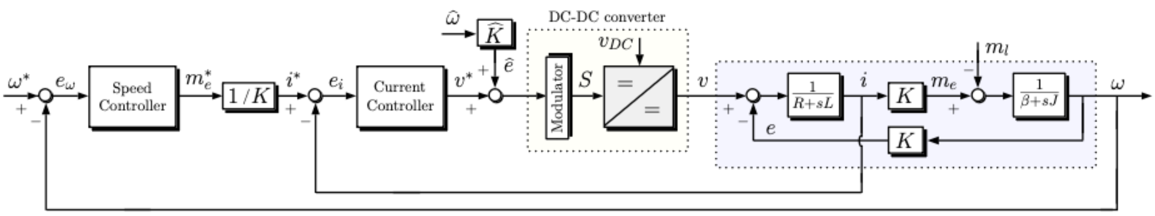


Figure 3 - Decoupling

This modelling approach is highly versatile. To convert the PM DC model to a Separately Excited (SE) DC motor model, the parameters are updated as follows:

- $R \rightarrow R_a$
- $L \rightarrow L_a$
- $K \rightarrow K * I_e$

3.2 Mechanical model

The model can be described as one wheel with radius R and mass M rolling on a slope under no-slip condition.
The goal is to compute the mechanical load applied to the system.
In figure 3 is shown the forces diagram:

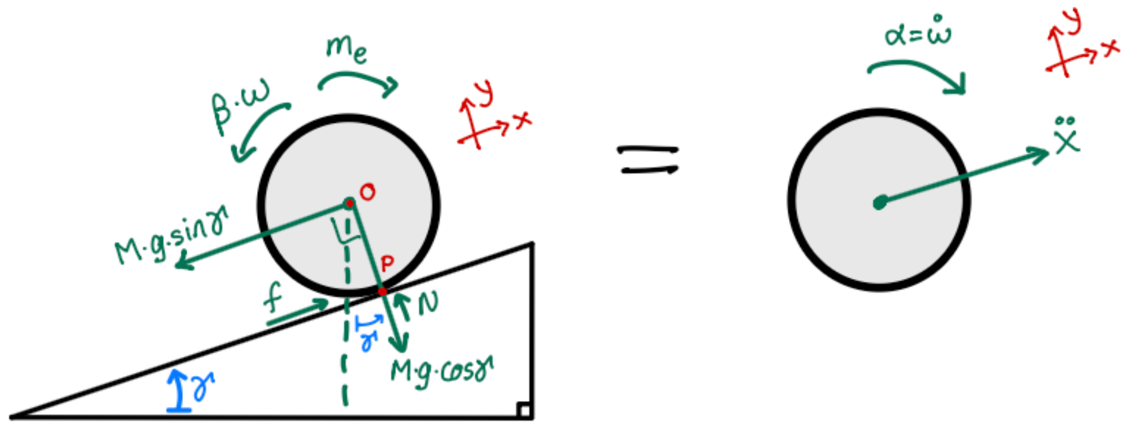


Figure 4 - Mechanical model

Under the no-slip condition $v = r \cdot \omega_w$, where $r = \frac{d}{2}$ is the wheel radius and ω_w the wheel angular velocity, and assuming a rigid transmission with ratio $\rho = \frac{\omega_w}{\omega_m}$, the vehicle linear velocity is:

$$v = \rho \cdot r \cdot \omega_m$$

By the principle of virtual work, the translational mass M and the gravitational load can be reflected to the motor shaft. The resulting single-degree-of-freedom equation at the motor shaft is:

$$J_{eq} \cdot \frac{d\omega_m}{dt} + \beta \cdot \omega_m = m_e - T_{dist(\gamma)}$$

with:
- $J_{eq} = M \cdot (\rho \cdot r)^2$ — equivalent inertia
- $T_{dist(\gamma)} = M \cdot g \cdot sin(\gamma) \cdot \rho \cdot r$ — gravitational load torque referred to the shaft
- β — viscous friction coefficient, already referred to the motor shaft $[N \cdot m \cdot s]$

Disturbance analysis

The gravitational load torque referred to the motor shaft on a slope of grade s% is:

$$T\_dist = M * g * sin\left(arctan\left(\frac{s}{100}\right)\right) * rho * \left(\frac{d}{2}\right)$$

For s = 5% and M = 25 400 kg:

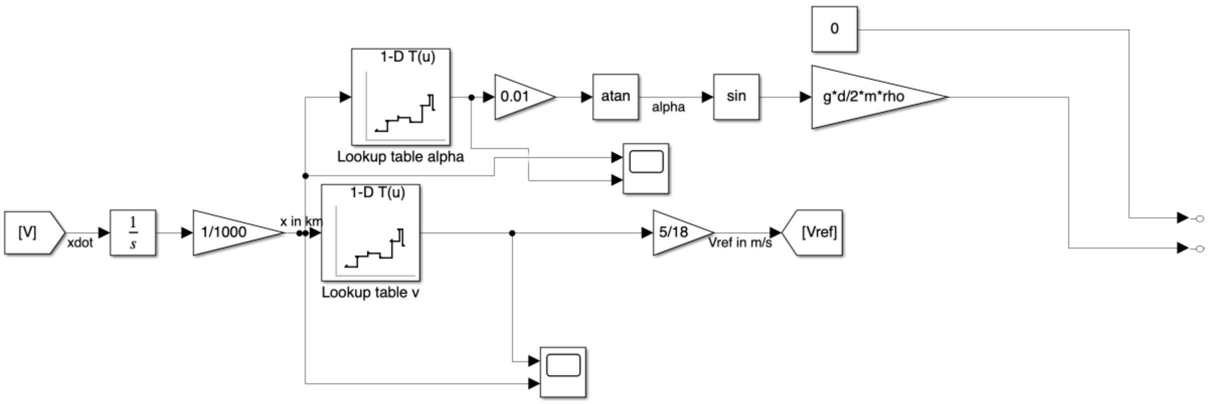
$$T_{dist} = 25400 * 9.81 * sin(arctan(0.05)) * \left(\frac{13}{74}\right) * 0.34 = 743\, Nm$$

This represents 89.8% of the rated torque T_n. The speed PI controller must therefore reject a disturbance nearly equal to the full rated torque.
The integrator action of the PI guarantees zero steady-state speed error under a constant torque disturbance; however, the transient speed dip during slope entry depends on the loop bandwidth $omega_m = 2\, rad/s$.
The settling time is approximately $tau_s = \frac{4}{omega_m} = 2\, s$, during which a temporary speed deviation is expected and visible in the simulation results.

3.3 Simulink scheme of the model

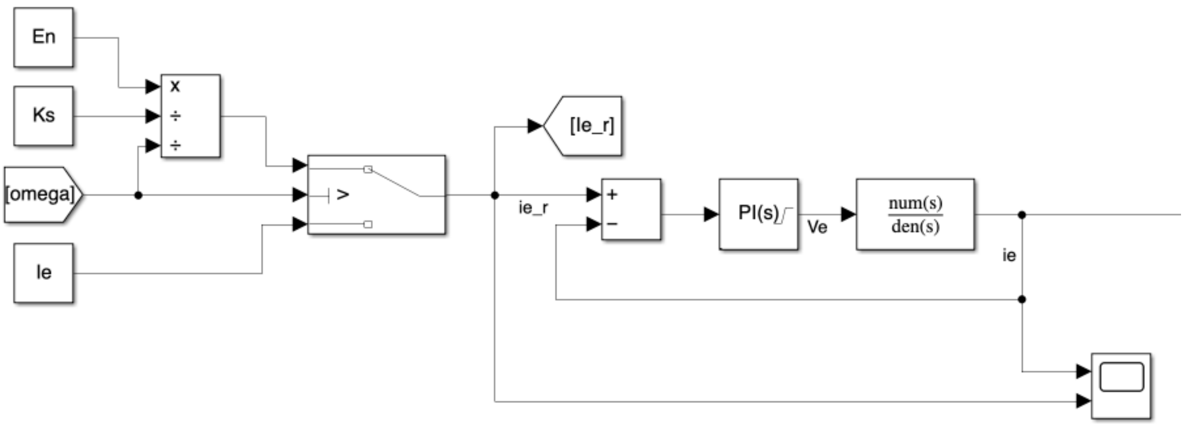
Mechanical control

Simulink block diagram of the mechanical control subsystem: lookup tables for slope angle and speed reference generation, with position integration and km conversion.



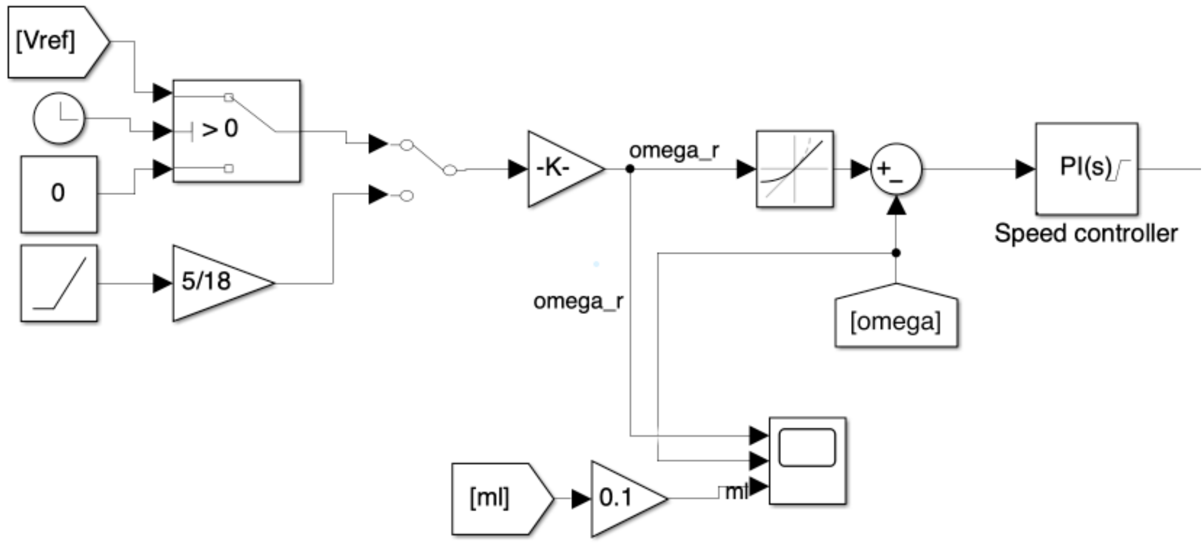
Excitation current control

Simulink diagram of the excitation current control loop with field-weakening logic: the reference ie_r is computed from En, Ks, omega, and Ie, then regulated by a PI controller driving the excitation plant Ge(s).



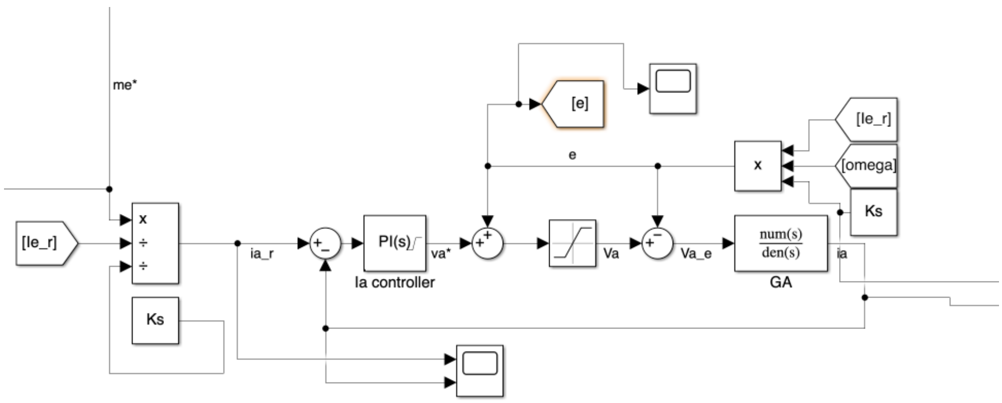
Torque reference

Simulink diagram of the speed reference and torque generation block: Vref selector (motor/regenerative), rate limiter, omega conversion, and speed PI controller input stage.



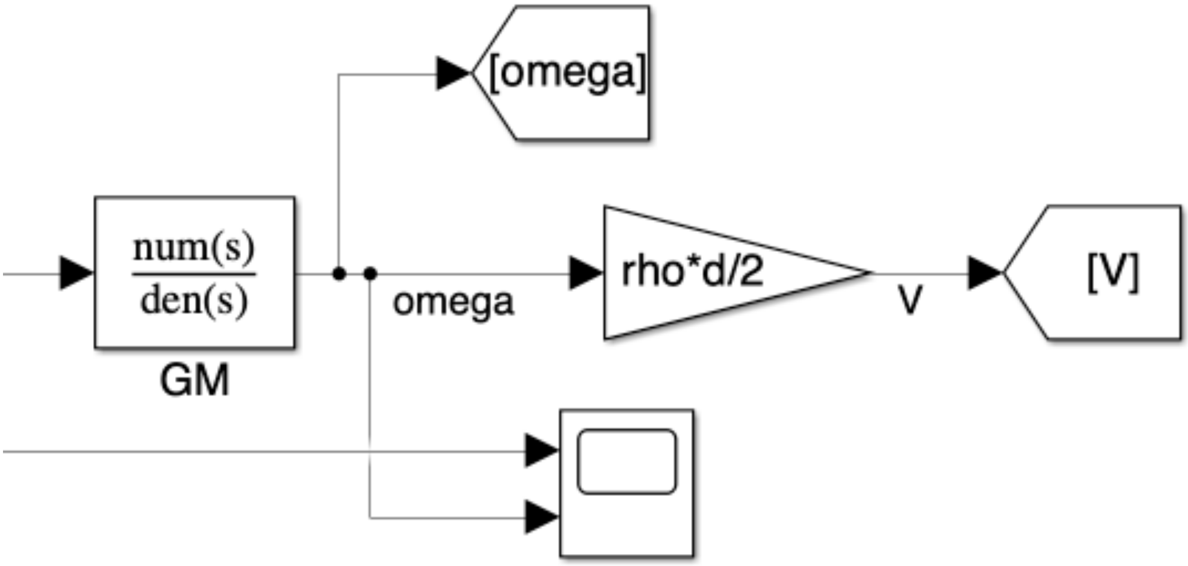
Armature current control

Simulink armature current control loop: ia reference computed from Te and Ie_r, regulated by a PI controller with voltage saturation Va and back-EMF decoupling e = Ks·Ie_r·omega, feeding the armature plant GA(s).



Speed control

Simulink mechanical plant subsystem: speed signal from GM(s), converted to vehicle linear velocity V via the kinematic gain ρ·d/2.



3.4 Parameters identification

Analysing the provided case study is possible to compute the required transfer function needed for the complete system:

1. Armature inductance: $L_a = R_a * \tau_a$
2. Excitation inductance: $L_e = R_e * \tau_e$
3. $K_s = K_t$
4. $E_n = \Omega_n * I_e * K_s$
5. Total mass of the vehicle: $M = m_t + m_p * n_p$

The transfer functions of the system can be described as follow. All the transfer functions are asymptotically stable:

- Armature current transfer function:

$$G_a(s) = \frac{1}{R_a + sL_a}$$

- Excitation current transfer function:

$$G_e(s) = \frac{1}{R_e + sL_e}$$

- Mechanical transfer function:

$$G_\omega(s) = \frac{1}{\beta + sJ}$$

4. Control design

The system can be controlled by building a cascade control system with three controllers.
To control the system is used a PI controller because a derivative action (PID) will increase the complexity of the controller and creates overshoot.

4.1 PI controller

The system requires phase margin 90° to give stability at the system.

Controller	Bandwidth	Phase margin
Excitation current	40	90°
Armature current	20	90°
Angular speed	2	90°

The parameters of the PI controller are computed in MATLAB:

Controller	*Bandwidth*	*Phase margin*	*Kp*	*Ki*
Excitation current	40	90°	48	480
Armature current	20	90°	0,078	7,800
Angular speed	2	90°	181,2	1,620

The ratio $\frac{omega_i}{omega_m} = 10$ is at the theoretical lower bound. This choice is deliberate: a lower speed bandwidth omega_m would produce excessively long settling times ($tau_s \sim \frac{4}{omega_m}$) on the 10 km route, degrading tracking performance. The approximation error introduced by the borderline separation is acceptable given that the inner current loop reaches steady state in < 0.5 s, well before the speed loop transient completes.
The simulation results confirm adequate decoupling.

4.2 Limitations

There are limitations related to the actuators due to the design of the motor. Because of this limitation is needed a saturation to replicate the system.
To resolve the issue is used a control strategy called "Antiwindup" schema.

The schema is using back calculation when the system goes into saturation. The schema is provided as follow:

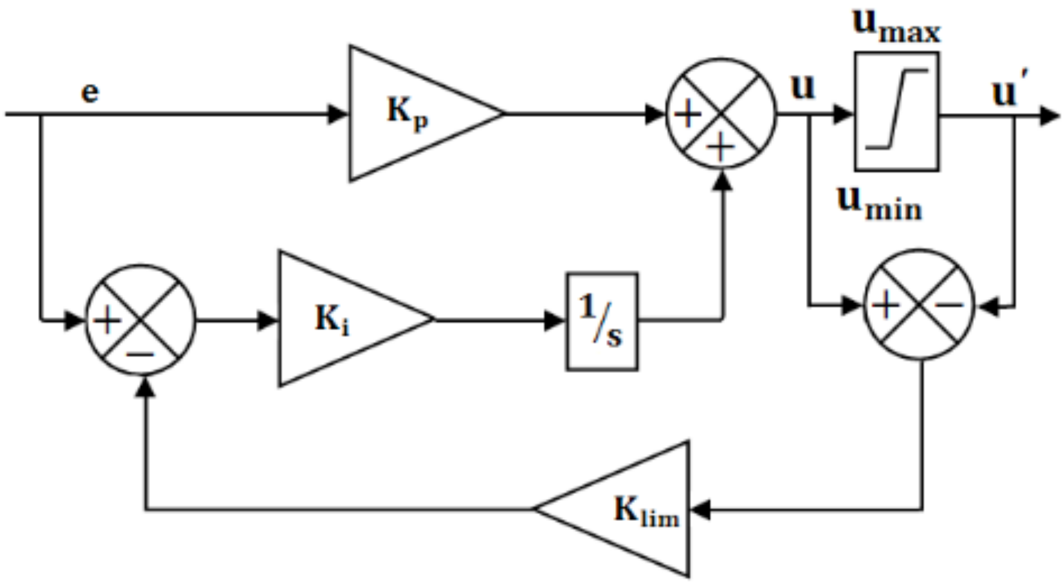


Figure 5 - Antiwindup schema

To compute back calculation is needed the parameter $K_b = \frac{1}{\tau}$ for each loop.

Controller	τ	K_{lim}
Excitation current	0,1	10
Armature current	0,01	100
Angular speed	111,87	0,0089

Back calculation is the most used schema in industrial application due to is simplicity, good transitory response and is robustness.

*4.3 **Rate limiter***

The speed reference generated from the kinematic lookup table consists of a sequence of step changes corresponding to the different speed segments along the 10 km route. If applied directly to the speed PI controller, these abrupt step changes would demand an instantaneous torque impulse, driving the armature current deep into saturation and imposing unacceptable jerks on passengers and mechanical components.

To overcome this issue, a Rate Limiter block is inserted on the angular speed reference wire ω_{ref} immediately before the speed PI controller summing junction. The block replaces every step transition with a linear ramp, enforcing a constant-acceleration (or constant-deceleration) profile that keeps passenger comfort within acceptable bounds.

4.3.1 Design and implementation

The maximum admissible linear acceleration for a standing passenger in public urban transport is conventionally set at $a_{max} = 0{,}545\ m/s^2$.
Converting this to the motor shaft through the kinematic chain (gearbox ratio $\rho = \frac{13}{74}$ and wheel radius $r = \frac{d}{2} = 0{,}34\ m$) yields:

$$\alpha_{max} = \frac{a_{max}}{\rho * \frac{d}{2}} = 9{,}13\,\frac{rad}{s^2}$$

5. Results

This section presents and discusses the three key simulation outputs: vehicle speed, armature current, and excitation current.

5.1 Speed control

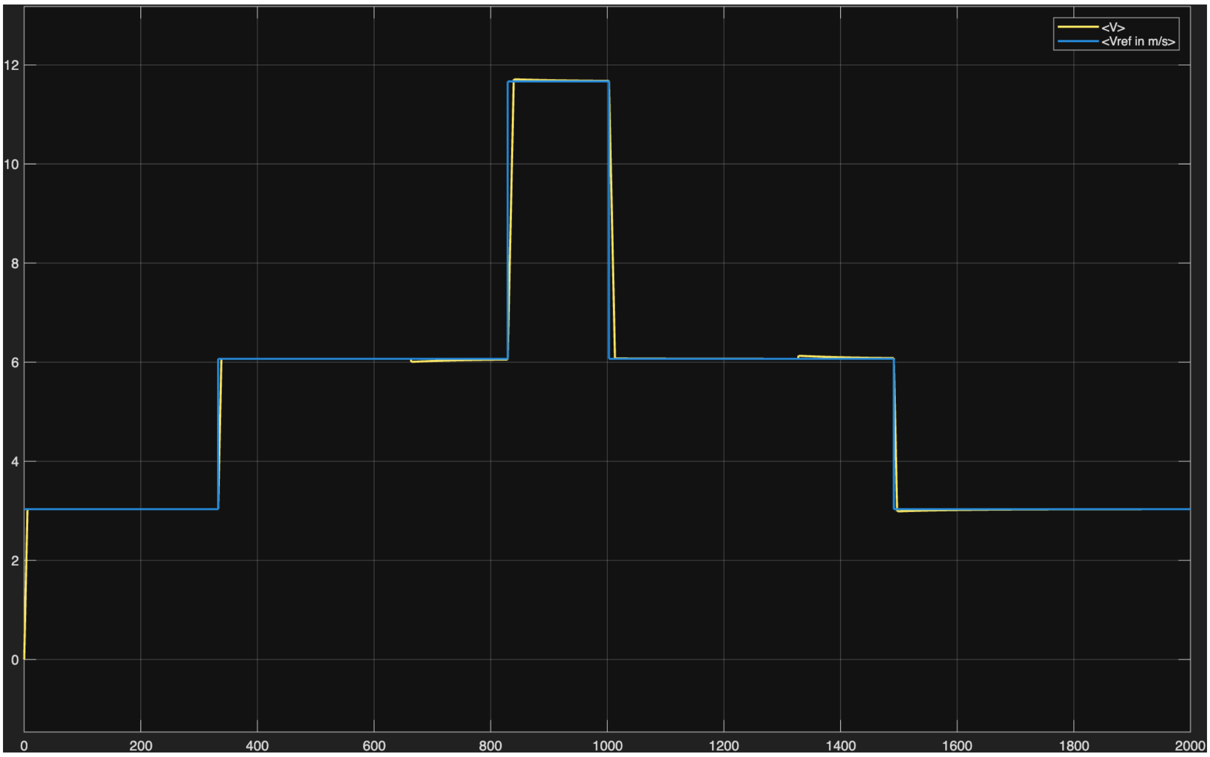


Figure 6 – Speed

The figure illustrates the speed profile of the tramway during the simulation. The linear ramp indicates a constant acceleration phase where the PI controller accurately tracks the reference. The absence of overshoot confirms the correct tuning of the speed loop through the pole-zero cancellation method, ensuring passenger comfort and mechanical integrity.

Disturbance rejection on graded segments

At km 3–4 (uphill, +5%) the gravitational disturbance torque reaches $T_{dist} = 743\ Nm$ corresponding to 89.8% of the rated torque $T_n = 827\ Nm$.

The speed PI controller rejects this constant disturbance through its integral action, restoring zero steady-state speed error.

The gravitational component alone requires approximately 140 A. Including viscous friction at rated speed, the total required current approaches the rated limit of 156 A.

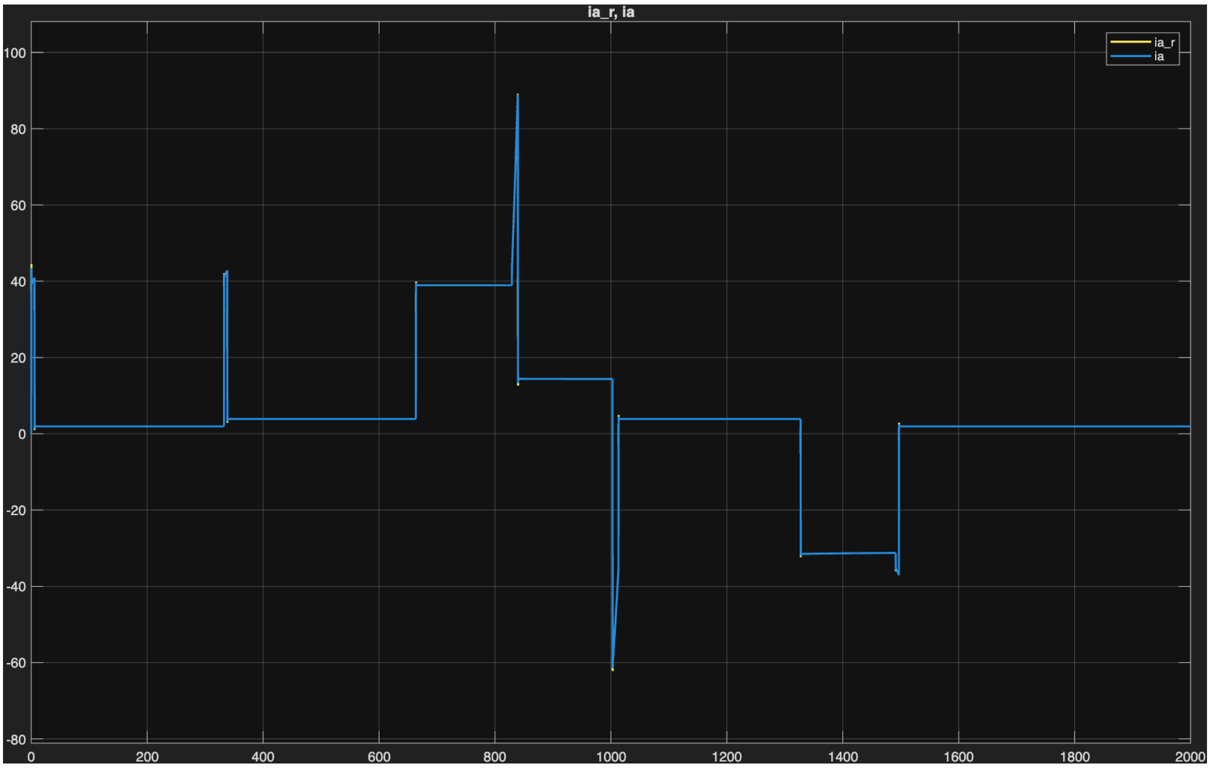
The armature current reverses sign, and energy is returned to the 600 V DC line.

The anti-windup scheme ensures smooth transitions at the saturation boundaries.

Key observations:

- Zero steady-state error and no overshoot: once the reference is reached, the PI controller maintains the speed with negligible error. The absence of overshoot validates the pole-zero cancellation design and the 90° phase margin target.
- Field-weakening region (km 4–6, $V_{max} \approx 11.67$ m/s): the controller successfully drives the vehicle to maximum speed. The speed is tracked accurately despite the excitation current being reduced below its nominal value.

5.2 *Armature current control*

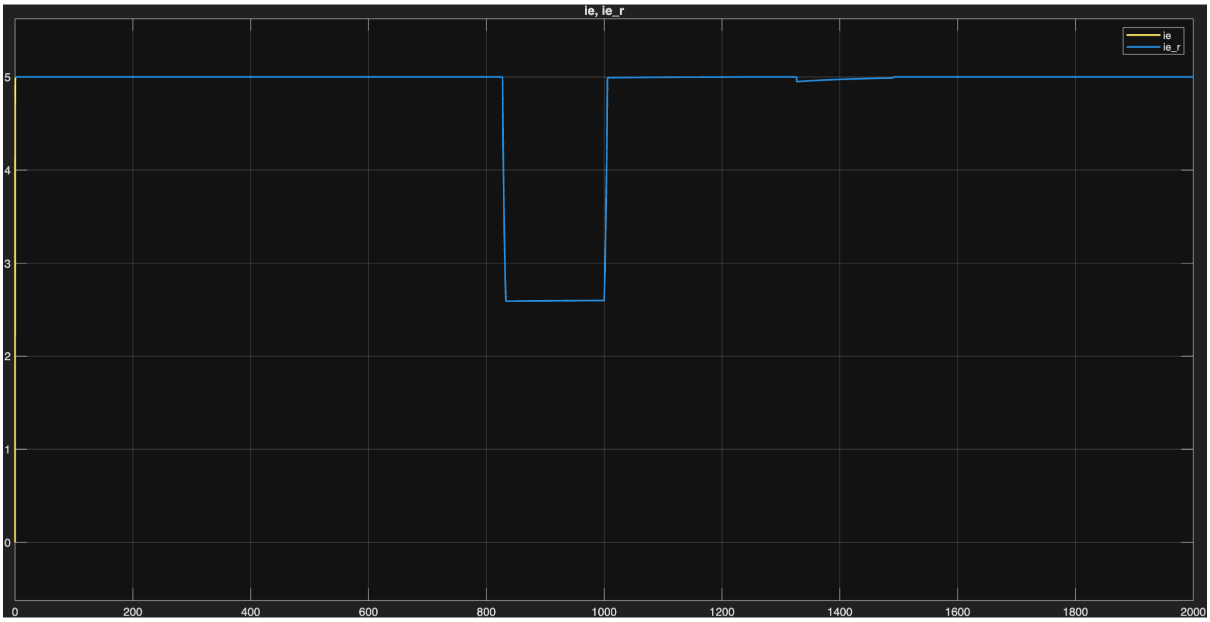


The armature current is strictly limited to the rated value by the controller's saturation block to prevent thermal stress on the motor windings. Thanks to the implemented back-calculation anti-windup scheme, the transition from acceleration to constant speed is smooth, without any current spikes that could trip the electrical protection systems.

Key observations:

- Saturation at rated current during acceleration: whenever the Rate Limiter demands maximum torque, I_a is clamped at +156 A by the saturation block. This maximises traction effort while protecting the motor windings from thermal overload.
- Reduced steady-state current at constant speed: outside of ramp phases, the torque required only needs to overcome viscous friction and slope disturbance. The armature current settles to a low steady-state value, confirming efficient operation.

5.3 *Excitation current control*



The excitation current plot validates the field-weakening strategy. The current remains at its nominal value of 5 A until the base speed is reached. Beyond this point, the controller dynamically reduces the excitation following a $\frac{1}{\Omega}$ characteristic.
This reduction is vital to keeping the back-e.m.f. below the 600 V threshold, allowing the motor to operate in the constant-power region.
Key observations:

- Constant excitation in base-speed region: throughout the km 0–3 and km 6–10 segments, I_e is held at its nominal value of 5 A. The excitation PI controller tracks the constant reference without error, confirming a phase margin of 90° and bandwidth ω^e = 40 rad/s.
- Field-weakening activation at V_{max}: as the speed reference exceeds, the field-weakening block computes $I_{e_{ref}} = \frac{E_n}{K_s \cdot \omega}$. At maximum speed, the reference drops to $I_{e_{ref}} \approx 2.6\ A$. The excitation PI tracks this reduction accurately and without oscillation.
- Return to nominal excitation: when the speed drops back to V_r, the field-weakening block restores $I_{e_{ref}} = 5\ A$.

Field-weakening constraint verification

At maximum speed $omega_{max} \ = \ 195.3 \frac{rad}{s}$, the field-weakening law gives:

$$i_{e_{ref}}(omega_{max}) = \frac{539.1}{1.06 \ * \ 195.3} = \ 2.60 \ A$$

The resulting back-EMF is:

$$e \ = \ K_S * \ i_{e_{ref}} * omega_{max} = 1.06 * 2.60 * 195.3 = 538.8 \ V < \ 600 \ V$$

The back-EMF remains 0.05% below the rated value $E_n \ = \ 539.1 \ V$ across the entire field-weakening range, confirming that the 600 V line limit is never violated.

The field-weakening speed ratio is $\frac{omega_{max}}{omega_{base}} = \frac{195.3}{101.7} = 1.92$, meaning the constant-power region spans a factor of approximately 1.9 in speed.

6. Conclusions

The analysis of the tramway case study demonstrates that a robust control architecture is essential for high-performance traction systems.
By implementing a dynamic field-weakening strategy, the drive successfully met all operational requirements while strictly adhering to electrical constraints.
The study highlights the engineer's responsibility to balance control complexity with physical system limitations.
Ultimately, the correct integration of power-constant operation and anti-windup schemes ensures that safety-critical environments like public transport achieve superior standards of operational efficiency and passenger comfort.
All three control loops: excitation current, armature current, and angular speed perform within specification.
The Rate Limiter, anti-windup back-calculation, and field-weakening strategy work in concert to deliver safe, comfortable, and energy-efficient traction control across the entire route, including graded and maximum-speed segments.

The disturbance rejection analysis confirms that the speed PI controller successfully rejects gravitational disturbance torques of up to 743 Nm (89.8% of rated torque) on the 5% grade segments, with zero steady-state speed error guaranteed by the integral action.
The field-weakening strategy maintains the back-EMF within 0.05% of $E_n = 539\,V$ across the entire constant-power operating region ($omega_{base}$ to $omega_{max}$ $=$ 101,7 to 195,3 $rad/$s, ratio 1,92), strictly respecting the 600 V line voltage constraint at all operating points.

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7. MATLAB code

```
clc; clear; close all;

%  CONSISTENCY CHECK (pre-verification, run mentally before simulation)
%
%    En_motor  = Ks * Ie * omega_n  =  1.06 * 5 * 101.6  ≈  539 V
%    En_KVL    = V  - Ra * Ia       =  600  - 0.39 * 156  ≈  539 V
%    Tn_motor  = Ks * Ie * Ia       =  1.06 * 5 * 156     ≈  827 Nm
%    Tn_mech   = 4 * Pn / omega_n   =  84000 / 101.6      ≈  827 Nm
% -----------------------------------------------------------------------


g = 9.81;           % Gravitational acceleration          [m/s^2]


%% ====================================================================
%  2.  MOTOR PARAMETERS  (equivalent model for 4 motors in parallel)

V      = 600;       % DC line voltage                     [V]

% --- Armature circuit ------------------------------------------------
Pn     = 21e3;      % Rated power per motor               [W]
Imaxn  = 156;       % Total rated armature current        [A]
Ra     = 0.39;      % Equivalent armature resistance      [Ohm]
ta     = 10e-3;     % Armature circuit time constant      [s]
La     = ta * Ra;   % Equivalent armature inductance      [H]

% --- Excitation circuit (same for every motor) -----------------------
Ve     = 60;        % Rated excitation voltage            [V]
Ie     = 5;         % Rated excitation current            [A]
Re     = 12;        % Excitation resistance               [Ohm]
te     = 0.1;       % Excitation time constant            [s]
Le     = te * Re;   % Excitation inductance               [H]

% --- Machine constant ------------------------------------------------
%  Kt is interpreted as the TOTAL machine constant Ks (see Section 3
%  of the report for the KVL-based justification).

Kt     = 1.06;      % Machine constant from datasheet     [Nm/A^2]
Ks     = Kt;        % Total machine constant  Ks = Kt     [Nm/A^2]

% --- Rated speed -----------------------------------------------------
omen   = 970;                   % Rated speed              [rpm]
ome    = omen * 2 * pi / 60;    % Rated speed              [rad/s]

% --- Maximum vehicle speed -------------------------------------------
v_max_ms = 42 / 3.6;            % Maximum vehicle speed    [m/s]
```

```
%% ======================================================================

%  3.  PARAMETER CONSISTENCY VERIFICATION

En_motor   = Ks * Ie * ome;             % Back-EMF from machine constant
[V]
En_KVL     = V  - Ra * Imaxn;           % Back-EMF from Kirchhoff's law
[V]
Tn_motor   = Ks * Ie * Imaxn;          % Rated torque – machine equation
[Nm]
Tn_mech    = 4  * Pn / ome;            % Rated torque – power balance
[Nm]
eta        = (4 * Pn) / (V * Imaxn);   % Nominal efficiency
[-]
En         = En_motor;                  % Nominal back-EMF used hereafter
[V]

fprintf('=== PARAMETER CONSISTENCY CHECK ===\n');
fprintf('  En  (Ks·Ie·ωn)  : %8.2f V\n',  En_motor);
fprintf('  En  (V – Ra·Ia) : %8.2f V\n',  En_KVL);
fprintf('  Relative error  : %8.3f %%\n', abs(En_motor - En_KVL) / En_KVL
* 100);
fprintf('  ---\n');
fprintf('  Tn  (Ks·Ie·Ia)  : %8.1f Nm\n', Tn_motor);
fprintf('  Tn  (4·Pn/ω)    : %8.1f Nm\n', Tn_mech);
fprintf('  Relative error  : %8.3f %%\n', abs(Tn_motor - Tn_mech) /
Tn_mech * 100);
fprintf('  ---\n');
fprintf('  Nominal efficiency : %.3f\n',   eta);
fprintf('====================================\n\n');


%% ======================================================================
%  4.  MECHANICAL PARAMETERS  (referred to the motor shaft)

mt      = 15e3;          % Vehicle mass – unloaded                  [kg]
mp      = 80;            % Average passenger mass                   [kg]
np      = 130;           % Maximum number of passengers             [-]
m       = mt + mp * np;  % Total vehicle mass  (= 25 400 kg)        [kg]

d       = 680e-3;        % Wheel diameter                           [m]
rho     = 13 / 74;       % Gearbox ratio  (motor shaft → wheel)     [-]
beta    = 0.81;          % Viscous friction coefficient             [Nms]

% Equivalent moment of inertia referred to the motor shaft.
% Assumption: rigid transmission; rotor / gear inertia neglected.
J = m * (rho * d / 2)^2;   % Equivalent inertia
[kg·m^2]

fprintf('=== MECHANICAL PARAMETERS ===\n');
fprintf('  Total mass   m : %8.0f kg\n',     m);
fprintf('  Equiv. inertia J : %8.4f kg·m^2\n', J);
fprintf('=============================\n\n');
```

```
%% ========================================================================
%  5.  TRANSFER FUNCTION PARAMETERS

tG_a = ta;       G_a = 1 / Ra;    % Armature :   Ga(s) = G_a / (1 +
s·tG_a)
tG_e = te;       G_e = 1 / Re;    % Excitation : Ge(s) = G_e / (1 +
s·tG_e)
tGm  = J / beta; Gm  = 1 / beta;  % Mechanical : Gm(s) = Gm  / (1 + s·tGm
)

fprintf('=== PLANT TIME CONSTANTS ===\n');
fprintf('  tau_armature    : %8.4f s\n', tG_a);
fprintf('  tau_excitation  : %8.4f s\n', tG_e);
fprintf('  tau_mechanical  : %8.2f s\n', tGm);
fprintf('============================\n\n');


%% ========================================================================
%  6.  PI CONTROLLER DESIGN  —  pole-zero cancellation method

wi = 20;    % Armature current loop bandwidth    [rad/s]
we = 40;    % Excitation current loop bandwidth  [rad/s]
wm =  2;    % Angular speed loop bandwidth       [rad/s]

% PI gains:  Kp = omega_i · L,   Ki = omega_i · R
KpA = wi * La;      KiA = wi * Ra;      % Armature current controller
KpE = we * Le;      KiE = we * Re;      % Excitation current controller
KpM = wm * J;       KiM = wm * beta;    % Speed controller

fprintf('=== PI CONTROLLER GAINS ===\n');
fprintf('  Armature     — Kp = %8.4f   Ki = %8.4f\n', KpA, KiA);
fprintf('  Excitation   — Kp = %8.4f   Ki = %8.4f\n', KpE, KiE);
fprintf('  Speed        — Kp = %8.4f   Ki = %8.6f\n', KpM, KiM);
fprintf('===========================\n\n');


%% ========================================================================
%  7.  ANTI-WINDUP — back-calculation gains

Kb_a = 1 / ta;      % Armature  anti-windup gain  =  100    [1/s]
Kb_e = 1 / te;      % Excitation anti-windup gain =   10    [1/s]
Kb_m = 1 / tGm;     % Speed     anti-windup gain  ≈ 0.0089  [1/s]

fprintf('=== ANTI-WINDUP GAINS (back-calculation) ===\n');
fprintf('  Kb_armature    : %8.4f   (tau = %.4f s)\n', Kb_a, ta);
fprintf('  Kb_excitation  : %8.4f   (tau = %.4f s)\n', Kb_e, te);
fprintf('  Kb_speed       : %8.6f   (tau = %.2f  s)\n', Kb_m, tGm);
fprintf('============================================\n\n');

% Saturation limits for each loop
Te_max    =  Ks * Ie * Imaxn;  % Torque saturation on Speed PI output
[Nm]
Te_min    = -Te_max;           %
[Nm]
```

```
Ia_max    =  Imaxn;             % Current saturation on Ia PI output
[A]
Ia_min    = -Imaxn;             %
[A]
Va_max    =  V;                 % Voltage saturation on Va
[V]
Va_min    = -V;                 %
[V]

fprintf('=== SATURATION LIMITS ===\n');
fprintf('  Speed PI output  (Te) : [%+.1f , %+.1f] Nm\n', Te_min, Te_max);
fprintf('  Armature PI output (Va): [%+.1f , %+.1f] V\n',  Va_min,
Va_max);
fprintf('=========================\n\n');


%% ======================================================================
%  8.  FIELD WEAKENING  —  constant-power operating region

ome_base   = En / (Ks * Ie);            % Base speed (start of FW region)
[rad/s]
ome_max    = v_max_ms / (rho * d / 2); % Maximum motor shaft speed
[rad/s]
Ie_at_vmax = Ie * ome_base / ome_max;  % Excitation current at v_max
[A]

fprintf('=== FIELD WEAKENING ===\n');
fprintf('  omega_rated      : %8.3f rad/s  (%5.1f rpm)\n', ome,
omen);
fprintf('  omega_base (FW)  : %8.3f rad/s  (= omega_rated — check)\n',
ome_base);
fprintf('  omega_max        : %8.3f rad/s  (%5.1f rpm)\n', ome_max,
ome_max * 60 / (2*pi));
fprintf('  FW speed ratio   : %8.3f  (v_max / v_base)\n', ome_max /
ome_base);
fprintf('  Ie at v_max      : %8.3f A\n', Ie_at_vmax);
fprintf('======================\n\n');


%% ======================================================================
%  9.  SLOPE DISTURBANCE VERIFICATION

gamma_5 = atan(0.05);   % Slope angle for ±5 % grade              [rad]

% Disturbance torque referred to the motor shaft
T_dist_up   =  m * g * sin(gamma_5) * rho * (d/2);  % Uphill   [Nm]
T_dist_down = -T_dist_up;                             % Downhill [Nm]

fprintf('=== SLOPE DISTURBANCE TORQUE ===\n');
fprintf('  T_dist  (+5 %% uphill)   : %8.1f Nm\n',  T_dist_up);
fprintf('  T_dist  (-5 %% downhill) : %8.1f Nm\n',  T_dist_down);
fprintf('  Rated torque  Tn        : %8.1f Nm\n',  Tn_motor);
fprintf('  Disturbance / Tn ratio  : %8.3f\n',     T_dist_up / Tn_motor);
fprintf('================================\n\n');
```

```
%% =======================================================================
%  10.  RATE LIMITER — speed reference profiling

a_max_comfort = 0.545;                          % Max acceleration (passenger
comfort) [m/s^2]
alpha_max     = a_max_comfort / (rho * d / 2);  % Motor shaft acceleration
[rad/s^2]

fprintf('=== RATE LIMITER (speed reference) ===\n');
fprintf('  Max linear acceleration  : %.2f m/s^2\n', a_max_comfort);
fprintf('  Max angular acceleration : %.4f rad/s^2  → use as slew rate\n',
alpha_max);
fprintf('=======================================\n\n');


%% =======================================================================
%  11.  SIMULINK VARIABLE SUMMARY

fprintf('=== WORKSPACE VARIABLES FOR SIMULINK ===\n');
fprintf('  Ks          = %.4f    Nm/A^2\n',  Ks);
fprintf('  En          = %.2f    V\n',        En);
fprintf('  La          = %.6f  H\n',         La);
fprintf('  Ra          = %.4f    Ohm\n',      Ra);
fprintf('  Le          = %.4f    H\n',        Le);
fprintf('  Re          = %.4f    Ohm\n',      Re);
fprintf('  J           = %.4f    kg·m^2\n',  J);
fprintf('  beta        = %.4f    Nms\n',      beta);
fprintf('  KpA         = %.6f\n',             KpA);
fprintf('  KiA         = %.4f\n',             KiA);
fprintf('  KpE         = %.4f\n',             KpE);
fprintf('  KiE         = %.4f\n',             KiE);
fprintf('  KpM         = %.4f\n',             KpM);
fprintf('  KiM         = %.6f\n',             KiM);
fprintf('  Kb_a        = %.4f\n',             Kb_a);
fprintf('  Kb_e        = %.4f\n',             Kb_e);
fprintf('  Kb_m        = %.8f\n',             Kb_m);
fprintf('  Te_max      = %.1f   Nm\n',        Te_max);
fprintf('  Ia_max      = %.1f   A\n',         Ia_max);
fprintf('  ome_base    = %.4f    rad/s\n',    ome_base);
fprintf('  ome_max     = %.4f    rad/s\n',    ome_max);
fprintf('  alpha_max   = %.4f    rad/s^2\n',  alpha_max);
fprintf('=========================================\n\n');
```

8. References

1. Course Material: Castelli Dezza, F. (2026). *Lecture notes and materials for "Dynamics of electrical machine and drives"*