SCUOLA DI INGEGNERIA INDUSTRIALE
E DELL'INFORMAZIONE

Course: Internet of Things - 5 CFU

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Homework – Exercise 3

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Abstract

The exercise considers a RFID system based on Dynamic Frame ALOHA with 4 tags (N) and asks to evaluate the overall collision resolution efficiency η for different values of the initial frame size r1 ∈ {1, 2, 3, 4, 5, 6}.

In Dynamic Frame ALOHA, each tag independently and uniformly selects one slot within a frame of size r. Slots chosen by exactly one tag are successes; slots chosen by two or more tags result in a collision, and the colliding tags form the backlog to be resolved in subsequent frames. After the first frame, the frame size is dynamically set equal to the current backlog size. The efficiency is defined as η = N/E [total slots consumed], where the expected total cost accounts for all frames needed until every tag is successfully decoded.

The resolution strategy is as follows: we first derived the expected resolution cost L_4 for N = 4 tags by computing, via combinatorial analysis, the probability distribution of the backlog after the first frame of size r = 4. This yields a recursive equation in L_4, which is solved analytically using the given values L_2 = 4 and L_3 = 51/8. Once L_4 is known, the same procedure is applied for each initial frame size r1: the backlog distribution after the first frame of size r1 is computed, and the expected total cost E [total slots | r1] is evaluated as the sum of r1 plus the expected residual cost. Finally, η(r1) = 4/E [total slots | r1] is plotted and the value of r1 that maximises efficiency is identified and discussed.

Deriving L_4

For N = 4 tags and frame size r = 4 (total outcomes: $4^4 = 256$), we enumerate all partition patterns and compute the resulting backlog distribution:

Partition	Tag grouping	Backlog b	Outcomes	P(b | r=4)
{1,1,1,1}	all succeed	0	4! = 24	3/32
{2,1,1}	1 pair + 2 singles	2	144	9/16
{3,1}	1 triple + 1 single	3	48	3/16
{2,2} + {4}	2 pairs / all collide	4	36 + 4 = 40	5/32

The recursive equation for L_4 follows from the law of total expectation:

$$L_4 = 4 + (9/16)*L_2 + (3/16)*L_3 + (5/32)*L_4$$

Substituting $L_2 = 4$ and $L_3 = 51/8$:

$$L_4*(1 - 5/32) = 4 + 9/4 + 153/128 = 953/128$$
$$L_4*(27/32) = 953/128 \implies L_4 = 953/108 \approx 8.824 \text{ slots}$$

Efficiency η for each r_1

For each initial frame size r_1, the backlog distribution is computed combinatorially, and the expected total slots is evaluated as:

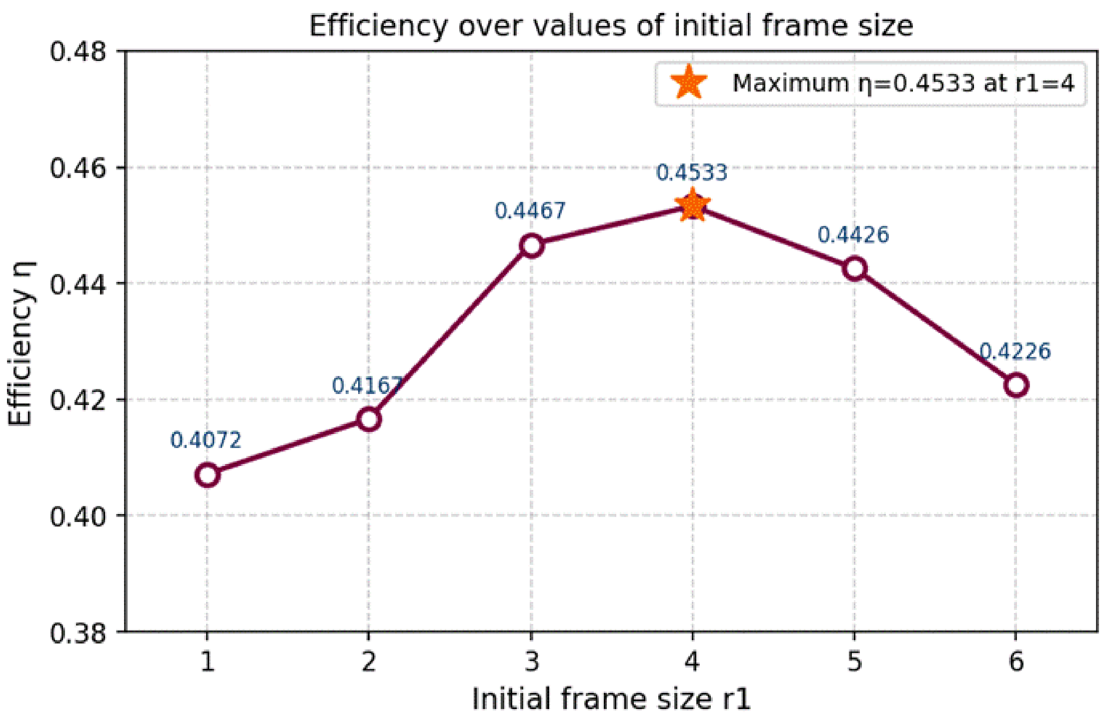
$$E[total \mid r_1] = r_1 + \Sigma_n P(b \mid r_1) \cdot L_n$$

r_1	E[total slots] – exact fraction	E[total slots] (approx.)	η = 4 / E
1	1061/108	9.8241	0.4072
2	4147/432	9.5995	0.4167
3	26111/2916	8.9544	0.4467

4 *	**953/108**	**8.8241**	**0.4533 ★ max**
5	122009/13500	9.0377	0.4426
6	110413/11664	9.4661	0.4226

** Row highlighted in gold = maximum efficiency, at r_1 = 4 = N.*

Plot: η vs. Initial Frame Size r_1



Maximum Efficiency and Optimal r_1

The maximum efficiency **η ≈ 0.4533** is achieved at **r_1 = 4 = N**. This is consistent with the well-known optimality condition for Frame Slotted ALOHA: the optimal initial frame size equals the number of contending tags. The reasoning proceeds as follows:

- **r_1 < N (undersized frame):** multiple tags are forced to collide in the same slots, creating large backlogs. The overhead from collision cascades grows rapidly as r_1 decreases below N.
- **r_1 = N (optimal):** the probability of each tag picking a unique slot is maximised, the expected backlog after the first frame is minimised, and therefore so is the total slot consumption over the entire resolution process.

- **$r_1 > N$ (oversized frame):** many slots are idle (no tag selects them), even if collisions are resolved quickly. Empty-slot waste increases with r_1, progressively reducing efficiency.

The efficiency curve confirms the expected **bell shape**, peaking at $r_1 = N = 4$ and degrading on either side. The degradation is **asymmetric**: efficiency drops more steeply for $r_1 < N$ (collision cascades amplify cost) than for $r_1 > N$ (only additive idle-slot waste), as visible in the plot above.